Inference at $*$ 1 0
of proof for Lemma neg_mul_arg_bounds:

1. a : $\mathbb{Z}$
2. b : $\mathbb{Z}$
3. $(((\text{-}a) * b) > 0) \iff ((((\text{-}a) > 0)\ \&\ (b > 0)) \vee (((\text{-}a) < 0)\ \&\ (b < 0)))$

$\vdash ((a * b) < 0) \iff (((a < 0)\ \&\ (b > 0)) \vee ((a > 0)\ \&\ (b < 0)))$

by PERMUTE{1:n,
2:n,
3:n,
4:n,
5:n,
4:n,
6:n,
7:n,
8:n,
9:n,
10:n,
11:n,
8:n,
9:n,
10:n,
12:n,
11:n,
8:n,
9:n,
13:n,
10:n,
14:n,
15:n}

1: .....wf..... NILNIL

$\vdash ((\text{-}a) * b) \in \mathbb{Z}$

2: .....wf..... NILNIL

$\vdash (\text{-}(a * b)) \in \mathbb{Z}$

3: .....wf..... NILNIL

$\vdash 0 = 0$

4: .....antecedent..... NILNIL

$\vdash$ True

5: .....wf..... NILNIL

$\vdash ((((\text{-}a) > 0)\ \&\ (b > 0)) \vee (((\text{-}a) < 0)\ \&\ (b < 0)))$
$=$
$((((\text{-}a) > 0)\ \&\ (b > 0)) \vee (((\text{-}a) < 0)\ \&\ (b < 0)))$
6: .....wf..... NILNIL

4. $((((\text{-}a) * b) > 0) \iff ((((\text{-}a) > 0)\ \&\ (b > 0)) \vee (((\text{-}a) < 0)\ \&\ (b < 0))))$
$=$
$(((\text{-}(a * b)) > 0) \iff ((((\text{-}a) > 0)\ \&\ (b > 0)) \vee (((\text{-}a) < 0)\ \&\ (b < 0))))$
$\vdash \mathbb{P}_1 = \mathbb{P}_1$
7: .....wf..... NILNIL

3. $((\text{-}(a * b)) > 0) \iff ((((\text{-}a) > 0)\ \&\ (b > 0)) \vee (((\text{-}a) < 0)\ \&\ (b < 0)))$
$\vdash (\text{-}(a * b)) = (\text{-}(a * b))$
8: .....wf..... NILNIL

3. $((\text{-}(a * b)) > 0) \iff ((((\text{-}a) > 0)\ \&\ (b > 0)) \vee (((\text{-}a) < 0)\ \&\ (b < 0)))$
$\vdash (\text{-}0) \in \mathbb{Z}$
9: .....wf..... NILNIL

3. $((\text{-}(a * b)) > 0) \iff ((((\text{-}a) > 0)\ \&\ (b > 0)) \vee (((\text{-}a) < 0)\ \&\ (b < 0)))$
$\vdash 0 \in \mathbb{Z}$
10: .....antecedent..... NILNIL

3. $((\text{-}(a * b)) > 0) \iff ((((\text{-}a) > 0)\ \&\ (b > 0)) \vee (((\text{-}a) < 0)\ \&\ (b < 0)))$
$\vdash$ True
11: .....wf..... NILNIL

3. $((\text{-}(a * b)) > 0) \iff ((((\text{-}a) > 0)\ \&\ (b > 0)) \vee (((\text{-}a) < 0)\ \&\ (b < 0)))$
$\vdash (\text{-}a) = (\text{-}a)$
12: .....wf..... NILNIL

3. $((\text{-}(a * b)) > 0) \iff ((((\text{-}a) > 0)\ \&\ (b > 0)) \vee (((\text{-}a) < 0)\ \&\ (b < 0)))$
$\vdash (b > 0) = (b > 0)$
13: .....wf..... NILNIL

3. $((\text{-}(a * b)) > 0) \iff ((((\text{-}a) > 0)\ \&\ (b > 0)) \vee (((\text{-}a) < 0)\ \&\ (b < 0)))$
$\vdash (b < 0) = (b < 0)$
14: .....wf..... NILNIL

3. $((\text{-}(a * b)) > 0) \iff ((((\text{-}a) > 0)\ \&\ (b > 0)) \vee (((\text{-}a) < 0)\ \&\ (b < 0)))$
4. $(((\text{-}(a * b)) > (\text{-}0)) \iff ((((\text{-}a) > (\text{-}0))\ \&\ (b > 0)) \vee (((\text{-}a) < (\text{-}0))\ \&\ (b < 0))))$
$=$
$(((\text{-}(a * b)) > 0) \iff ((((\text{-}a) > 0)\ \&\ (b > 0)) \vee (((\text{-}a) < 0)\ \&\ (b < 0))))$
$\vdash \mathbb{P}_1 = \mathbb{P}_1$
15:

3. $((\text{-}(a * b)) > (\text{-}0)) \iff ((((\text{-}a) > (\text{-}0)) \,\&\, (b > 0)) \vee (((\text{-}a) < (\text{-}0)) \,\&\, (b < 0)))$
$\vdash ((a * b) < 0) \iff (((a < 0) \,\&\, (b > 0)) \vee ((a > 0) \,\&\, (b < 0)))$

.

http://www.nuprl.org/FDLcontent/p0_942988_/p121_74264_{neg_mul_arg_bounds}1_0.html